

Brane world corrections to scalar vacuum force in RSII-p

Román Linares^{1,*} Hugo A. Morales-Técotl^{1,2,†} and Omar Pedraza^{1,3‡}

¹*Departamento de Física, Universidad Autónoma Metropolitana Iztapalapa,*

San Rafael Atlixco 186, C.P. 09340, México D.F., México,

²*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México,*

A. Postal 70-543, México D.F., México, and

³*Centro de Investigación en Ciencia Aplicada y Tecnología Avanzada,*

Unidad Legaria, Instituto Politécnico Nacional,

Av. Legaria 694, C.P. 11500, México D.F., México.

Abstract

Vacuum force is an interesting low energy test for brane worlds due to its dependence on field's modes and its role in submillimeter gravity experiments. In this work we generalize a previous model example: the scalar field vacuum force between two parallel plates lying in the brane of a Randall-Sundrum scenario extended by p compact dimensions (RSII- p). Upon use of Green's function technique, for the massless scalar field, the 4D force is obtained from a zero mode while corrections turn out attractive and depend on the separation between plates as $l^{-(6+p)}$. For the massive scalar field a quasilocalized mode yields the 4D force with attractive corrections behaving like $l^{-(10+p)}$. Corrections are negligible w.r.t. 4D force for $AdS_{(5+p)}$ radius less than $\sim 10^{-6}\text{m}$. Although the $p = 0$ case is not physically viable due to the different behavior in regard to localization for the massless scalar and electromagnetic fields it yields an useful comparison between the dimensional regularization and Green's function techniques as we describe in the discussion.

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*Electronic address: lirr@xanum.uam.mx

†Electronic address: hugo@xanum.uam.mx

‡Electronic address: opedrazao@ipn.mx

I. INTRODUCTION

Spacetimes including more than three spatial dimensions have been studied since the first attempt of Nördstrom to unify electromagnetism and scalar gravity [1], and the subsequent ones of Kaluza [2] and Klein [3] to unify general relativity and electromagnetism. Later on the idea was put forward that our observable universe was constrained to a brane subspace of a higher dimensional spacetime [4, 5, 6] to try to solve problems like the cosmological constant one.

More recently, motivated by M theory [7, 8], and following phenomenological insights like trying to solve the hierarchy problem such an idea has been revived [9, 10, 11, 12, 13]. In these brane world models, to be consistent with particle physics and recent submillimeter gravity experiments [14, 15, 16], the Standard Model fields are trapped on the brane whereas gravity is spread out over the full higher dimensional spacetime. Although extra dimensions were originally considered to be small near the Planck length [3], in brane worlds they could be as large as say 1TeV^{-1} or even infinite [10, 12]. Their effects range from cosmology to particle physics and astrophysics (see e.g. [17, 18, 19, 20]).

Low energy tests on the other hand have remained largely unexplored based on the idea that acceptable brane world scenarios should be built to be compatible with them. In contraposition it has been recently advanced that subjecting brane world scenarios to low energy tests may shed light on their viability. Indeed already available experimental data can be used in this way. Hence high precision atomic and Casimir force experiments become relevant [21, 22, 23, 24, 25]. The vacuum force predicted by H.B.G. Casimir in 1948 [26] between neutral perfect conducting plates, which has been confirmed experimentally [27, 28, 29, 30, 31, 32] involves two aspects naturally appearing in the study of brane worlds, namely the mode structure of matter fields and the submillimeter length scale, of order $1\mu\text{m}$, for which it becomes noticeable. In 4D such a force has been understood by means of QED [33] and the modification of the modes due to the presence of the plates. Extensions to other fields, geometries, materials, boundaries and models have been also studied [33]. Regarding extra dimensions the Casimir force has been discussed in string theory [34, 35, 36, 37] and Randall-Sundrum models [38, 39, 40, 41, 42] as well as within inflationary brane world universe models [43] and dark matter [44, 45].

Notice that whereas in 4D flat spacetimes different techniques like dimensional regular-

ization and Green's function yield the same results [33] this is not obvious for curved brane world models. For instance the latter was adopted in [23] for RSII-1 ($p=1$) in contrast with [46] for RSII ($p=0$) based on dimensional regularization and it would be desirable to be able to compare them. Our analysis for p arbitrary will allow to do so.

The interest in RSII- p comes from its property of localizing not only scalar and gravity but also gauge fields whenever $p > 0$ [47, 48, 49]. It corresponds to a $(3 + p)$ -brane with p compact dimensions and positive tension κ , embedded in a $(5 + p)$ -spacetime whose metric are two patches of AdS_{5+p} of curvature radius κ^{-1}

$$ds_{5+p}^2 = e^{-2\kappa|y|} \left[\eta_{\mu\nu} dx^\mu dx^\nu - \sum_{j=1}^p R_j^2 d\theta_j^2 \right] - dy^2. \quad (1)$$

Here $\eta_{\mu\nu} = \text{diag}(+, -, -, -)$ is the 4D Minkowski metric, y is a coordinate for the non-compact dimension, θ_j 's $\in [0, 2\pi)$ with $j = 0, \dots, p$, denote the coordinates for the compact dimensions whereas R_j 's denote their size. Notice that the p compact dimensions are warped. This metric can be obtained as an asymptotic solution to the $(5 + p)$ Einstein equations with negative bulk cosmological constant and a $(3 + p)$ -brane with an appropriately tuned energy-momentum tensor [50, 51]. It turns out that the brane tension κ is determined by the $(5 + p)$ -dimensional Planck mass and cosmological constant.

We consider the standard setting of two parallel plates in 3-space implemented by setting to zero our scalar field on two planes in ordinary 3-space (Dirichlet boundary conditions). As for the extra dimensions we adopt conditions for the scalar field consistent with RSII- p , namely matching across the brane for the non compact one and periodic for the compact ones. Effectively then our “*plates*” are two parallel planes in ordinary three space stretching along extra dimensions. The corresponding Casimir force hence provides a generalization to brane worlds of that obtained by Ambjorn and Wolfram [52] for the case of arbitrary number of Euclidean dimensions. Technically our plates are $2+1+p$ dimensional embedded in an ambient space of dimension $3+1+p$ so the difference in dimension, or codimension, is 1. Similarly [52] considered $d-1$ dimensional plates embedded in d space.

Now as it was noticed in [23, 47, 53, 54] the scalar modes corresponding to the non compact dimension in RSII- p present different behavior depending on whether the scalar field is massive or not. This justifies our splitting in two separate sections for such two cases.

This paper is organized as follows. Section II summarizes the calculation of the vacuum

force in RSII- p based on Green's function. Then in Sec. III the force is given for the massive scalar field whereas Sec. IV is devoted to the massless case. Finally Sec. V contains the discussion of our results and further developments. We use units in which $\hbar = 1, c = 1$.

II. CASIMIR FORCE AND GREEN'S FUNCTION

In this section we describe the ingredients and the strategy to get the Casimir force for the scalar field within RSII- p using the Green's function. First we describe the origin of the scalar modes associated to every dimension, ordinary or extra, by describing the equations and boundary conditions they fulfill. They are then incorporated to form the Green's function which is in turn related to the vacuum energy momentum tensor of the scalar field. The discontinuity of the latter for either of the plates provides then the required vacuum force.

Mode structure. The starting point of our analysis is the action in $(5 + p)$ dimensions for a scalar field $\Phi(X)$ in the RSII- p metric (1), here denoted g_{MN} ,

$$S = \int d^4x \prod_{j=1}^p R_j d\theta_j dy \sqrt{-g} \left(\frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi - \frac{1}{2} m_{5+p}^2 \Phi^2 \right). \quad (2)$$

Here $X^M \equiv (x^\mu, \theta_i, y)$ denote the associated coordinates and $m_{(5+p)}$ is the mass of the field. The corresponding equation of motion for the scalar field is given by

$$e^{2\kappa|y|} \square_4 \Phi - e^{2\kappa|y|} \sum_{j=1}^p \frac{1}{R_j^2} \partial_{\theta_j}^2 \Phi - \frac{1}{\sqrt{-g}} \partial_y [\sqrt{-g} \partial_y \Phi] + m_{5+p}^2 \Phi = 0, \quad (3)$$

which separates through $\Phi(X) = \varphi(x) \prod_{j=1}^p \Theta_j(\theta_j) \psi(y)$ into

$$\left(\partial_{\theta_j}^2 + m_{\theta_j}^2 R_j^2 \right) \Theta_j(\theta_j) = 0, \quad j = 1, \dots, p \quad (4)$$

$$\left(\partial_y^2 - (4 + p) \kappa \operatorname{sgn}(y) \partial_y - m_{5+p}^2 + m^2 e^{2\kappa|y|} \right) \psi(y) = 0, \quad (5)$$

$$\left(\square_4 + \sum_{j=1}^p m_{\theta_j}^2 + m^2 \right) \varphi(x) = 0. \quad (6)$$

The $(p + 1)$ separation constants with units of mass, m_{θ_j} and m , correspond to the spectra of the modes for the compact and non compact dimensions, respectively. They give rise in turn to the effective mass, m_4 , of the 4D modes in (6) through

$$m_4^2 := \sum_{j=1}^p m_{\theta_j}^2 + m^2. \quad (7)$$

To find mode solutions to the above Eqs. we require boundary conditions that we next spell out.

Boundary conditions. We shall incorporate three types of boundary conditions: (a) To implement the presence of the plates in 3-space we simply set $\varphi(z = 0, l) = 0$. (b) To match the modes across the brane along the non compact dimension we impose $\psi(y = 0^+) = \psi(y = 0^-)$ and $\partial_y \psi(y = 0^+) = \partial_y \psi(y = 0^-)$. (c) To account for the compactness of the p dimensions we set $\Theta_{n_j}(\theta_j) = \Theta_{n_j}(\theta_j + 2\pi)$. Hereby we obtain explicitly the plates represented by two parallel planes in 3-space and stretching along the extra dimensions. Conditions (a) through (c) are used for both cases $m_{5+p} \neq 0$ and $m_{5+p} = 0$ which yield different spectra for the modes depending on the non compact dimensions; they are denoted $\psi_m(y)$, as described in sect. III and IV. As for ordinary 3-space and p compact dimensions the result is identical for the two cases. The modes in θ_j are:

$$\Theta_{n_j}(\theta_j) = \frac{1}{\sqrt{2\pi R_j}} e^{i n_j \theta_j} \quad \text{where} \quad n_j = m_{\theta_j} R_j \in \mathbb{Z}. \quad (8)$$

Therefore the contributions of the extra compact dimensions to m_4 , are given in terms of $m_{\theta_j}^2 = n_j^2 / R_j^2$. Rather than providing detailed modes for ordinary space its Green's function fulfilling the appropriate boundary conditions is next presented.

Green's function. We intend to determine the Green's function for Eq. (3) in terms of the modes (4) and (5) and the 4D Green's function corresponding to Eq. (6). As for the latter one considers separately the two regions: between plates ($0 < z < l$) and, say, to the right of the plate located at $z = l$. To do so let us split up the 3D position vector in the way $\vec{x} = (\vec{x}_\perp, z)$, where z denotes the coordinate orthogonal to the plates and \vec{x}_\perp denotes a 2D vector orthogonal to the z direction. Using this parametrization the 4D Green's function is given by (see for instance [33])

$$G_{4D}(x, x'; m_4^2) = \int \frac{d\omega}{2\pi} \frac{d^2 k_\perp}{(2\pi)^2} e^{-i\omega(t-t') + i\vec{k}_\perp \cdot (\vec{x}_\perp - \vec{x}'_\perp)} G(z, z'), \quad (9)$$

where m_4 fulfills (7) and

$$G(z, z') = \begin{cases} G_{\text{in}}(z, z') = -\frac{1}{\lambda \sin \lambda l} \sin \lambda z_{<} \sin \lambda (z_{>} - l), & 0 \leq z, z' \leq l, \\ G_{\text{out}}(z, z') = \frac{1}{\lambda} (\sin \lambda (z_{<} - l) e^{i\lambda(z_{>} - l)}), & l \leq z, z'. \end{cases} \quad (10)$$

Here $z_{>}(z_{<})$ represents the greater (lesser) of z and z' while the boundary conditions used between plates became $G(0, z') = G(l, z') = 0$ and $G(z, z') \sim e^{ikz}$, as $z \rightarrow \infty$, to the right of the plate $z = l$.

The Green's function for (3) becomes [23]

$$G_{(5+p)D} = \sum_{\{n\}} \int' dm \prod_{j=1}^p \Theta_{n_j}^*(\theta_j) \Theta_{n_j}(\theta'_j) \psi_m(y) \psi_m(y') G_{4D}(x, x'; m_4^2), \quad (11)$$

where $\{n\}$ denotes the set $\{n_1, n_2, \dots, n_p \mid n_1 \in \mathbb{Z}, \dots, n_p \in \mathbb{Z}\}$, the Θ_{n_j} 's are given by (8), the 4D Green's function by (9), and dm is the measure for the continuous eigenvalues m which to include the zero value requires an extra term as it is seen below, and hence the prime [9]. The ψ_m 's are given in next two sections.

Casimir force. The knowledge of the Green's function allows to obtain the force per unit area between the plates lying on ordinary space as follows. Notice that the force between the plates is obtained by integrating over coordinates “lateral” to the plates. In this case: $\vec{x}_\perp, y, \theta_j$ due to the fact that the normal-normal component of vacuum energy momentum tensor in $3+1+p$ spatial dimensions has physical units of force per unit of “volume” of $2+1+p$ space:

$$F = \int_0^A d\vec{x}_\perp \int_{-\infty}^{\infty} dy e^{-\kappa|y|(p+3)} \left[\prod_{j=1}^p \int_0^{2\pi} R d\theta_j \right] \left[\langle T_{zz}^{\text{in}} \rangle \Big|_{z=l, y=0} - \langle T_{zz}^{\text{out}} \rangle \Big|_{z=l, y=0} \right], \quad (12)$$

where A is the area of the planes forming the plates in 3-space so we are taking a chunk of plate of volume $A \left(\frac{2}{\kappa(3+p)} \right) (2\pi R)^p$, and we assume $R_j = R$, $j = 1, \dots, p$. As usual, $\langle \dots \rangle$ denotes vacuum expectation values and the Green's function are related to the normal-normal components of the vacuum energy momentum tensor through

$$\langle T_{zz}^{\text{in/out}} \rangle \Big|_{z=l, y=0} = \frac{1}{2i} \partial_z \partial_{z'} G_{(5+p)}^{\text{in/out}}(x, y, \theta; x', y', \theta') \Big|_{x_\perp \rightarrow x'_\perp, z \rightarrow z'=l, y \rightarrow y'=0, \theta_j \rightarrow \theta'_j}. \quad (13)$$

The labels in/out in the r.h.s. of (13) imply the use of (9) in (11). To proceed further on the details of the Casimir force we need to know the details of the spectrum of the modes for the non compact dimension and hence we separate the analysis depending on whether $m_{(5+p)}$ is zero or not.

III. MASSIVE SCALAR FIELD

In this section we consider $m_{5+p} \neq 0$. Subject to the above boundary conditions all modes are massive

$$\psi_m(y) = e^{\frac{4+p}{2}\kappa y} \sqrt{\frac{m}{2\kappa}} \left[a_m J_\gamma \left(\frac{m e^{\kappa y}}{\kappa} \right) + b_m N_\gamma \left(\frac{m e^{\kappa y}}{\kappa} \right) \right], \quad m > 0. \quad (14)$$

J_γ and N_γ are the Bessel and Neumann functions respectively. $\gamma = \sqrt{\left(\frac{4+p}{2}\right)^2 + \left(\frac{m_{5+p}}{\kappa}\right)^2}$ and the coefficients a_m and b_m are given by

$$a_m = -\frac{A_m}{\sqrt{1 + A_m^2}}, \quad b_m = \frac{1}{\sqrt{1 + A_m^2}}, \quad (15)$$

where

$$A_m = \frac{N_{\gamma-1}\left(\frac{m}{\kappa}\right) - \left(\gamma - \frac{4+p}{2}\right) \frac{\kappa}{m} N_\gamma\left(\frac{m}{\kappa}\right)}{J_{\gamma-1}\left(\frac{m}{\kappa}\right) - \left(\gamma - \frac{4+p}{2}\right) \frac{\kappa}{m} J_\gamma\left(\frac{m}{\kappa}\right)}. \quad (16)$$

At this point we can use the modes (14) into eqs. (11), (12) and (13) to find the Casimir force in the present case

$$f_T := \frac{F}{A} = \frac{2}{3+p} \sum_{\{n\}} \int \frac{dm}{\kappa} \psi_m^2(0) \left(\frac{1}{2i} \partial_z \partial_{z'} G_{4D}^{\text{in/out}}(x, x'; m_4^2) \right)_{z \rightarrow z'=l} \quad (17)$$

$$= \frac{2}{3+p} \sum_{\{n\}} \int \frac{dm}{\kappa} \psi_m^2(0) f_{4D}(m_4) \quad (18)$$

since $f_{4D}(m_4)$ is the standard Casimir force for a scalar field of mass m_4

$$f_{4D}(m_4) = -\frac{\hbar c}{32\pi^2 l^4} \int_{2lm_4}^{\infty} dx \frac{x^2 \sqrt{x^2 - 4l^2 m_4^2}}{e^x - 1}, \quad (19)$$

with value given by (7), this result says extra dimensional modes contribute individually as an admixture of massive modes corresponding to compact and non compact dimensions but weighted by the square of the wave function of the non compact modes at the brane.

To extract more physical information some approximations are in order. Although none of the modes (14) is localized on the brane in the regime $m, m_{5+p} \ll \kappa$ there exists a quasilocalized state corresponding to the complex eigenvalues $m = m_q - i\Delta$ with

$$m_q^2 = \frac{2+p}{4+p} m_{5+p}^2, \quad \text{and} \quad \frac{\Delta}{m_q} = \frac{(2+p)\pi}{2^{4+p} [\Gamma(\frac{4+p}{2})]^2} \left(\frac{m_q}{\kappa}\right)^{2+p}. \quad (20)$$

A quasilocalized state can decay into the continuum modes.

At low energies, on the brane, there are two regions of the spectrum which become relevant for which the small argument approximations for Bessel and Neumann functions can be used yielding

$$\psi_m^2(0) \approx \begin{cases} \frac{2+p}{2} \kappa \delta(m_q - m) & \text{for } m \sim m_q, \\ \frac{2(2+p)}{\pi} \frac{\kappa \Delta}{m_q^{4+p}} m^{5+p} & \text{for } m \ll m_q. \end{cases} \quad (21)$$

p	0	1	2	3	4
$I(p)$	1.37	5.51	25.08	127.25	711.32

TABLE I: Numerical value of the integral (24) for different number of extra compact dimensions p .

In accordance with the low energy approximation we are using we can assume $m \ll R^{-1}$ and therefore only the zero modes of the compact coordinates are relevant ($n_1 = \dots = n_p = 0$). The effective 4D Casimir force in this approximation is

$$f_{\text{eff}} \approx f_{\text{quasi}} + f_{\text{light}} = \frac{2+p}{3+p} f_{4D}(m_q) + f_{\text{light}}, \quad (22)$$

$$f_{\text{light}} := \frac{4(2+p)}{\pi(3+p)} \frac{\Delta}{m_q^{7+p}} \int_{m \ll m_q} m^{5+p} f_{4D}(m) dm.$$

The first term is proportional to the standard 4D Casimir force of a massive scalar field whose mass is m_q , Eq. (20), whereas the second term corresponds to the contribution of the light modes. Upon the change of variables $m = \alpha/l$ we obtain

$$f_{\text{light}} \approx \frac{1920(2+p)}{\pi^3(3+p)} \frac{\Delta}{m_q} \frac{1}{(m_q l)^{6+p}} f_{4D}(0) I(p), \quad (23)$$

$$I(p) \equiv \frac{1}{32\pi^2} \int_0^\infty \alpha^{5+p} \int_{2\alpha}^\infty dx \frac{x^2 \sqrt{x^2 - 4\alpha^2}}{e^x - 1}, \quad (24)$$

where $f_{4D}(0) = -\frac{\pi^2}{480} \frac{\hbar c}{l^4}$ represents the standard Casimir force of a massless scalar field. The numerical values for $I(p)$ are tabulated in table I, for p in the range $0, \dots, 4$.

Recalling the light scalar field approximation $m_q l \ll 1 \Rightarrow f_{4D}(m_q) \approx f_{4D}(0)$ and considering $f_{\text{light}} \ll \frac{2+p}{3+p} f_{4D}(0)$ in Eq. (22) one gets the lower bound $m_q (m_q l)^{6+p} / \Delta \gg (1920) I(p) / \pi^3$, or equivalently, $(\kappa l)^{2+p} (m_q l)^4 \gg 120(2+p) I(p) / \pi^2 2^p [\Gamma((4+p)/2)]^2$.

IV. MASSLESS SCALAR FIELD

Now we use $m_{5+p} = 0$. The allowed modes for the non compact dimension are now a massless zero mode localized on the brane

$$\psi_0 = \sqrt{\frac{(2+p)\kappa}{2}}, \quad (25)$$

which satisfies the normalization condition, $\int_{-\infty}^\infty dy e^{-2\kappa|y|} \psi_0^2$ and the massive modes have the form (14) by taking $m_{5+p} = 0$ thus getting $\gamma \rightarrow \gamma_0 = \frac{4+p}{2}$ and $A_m \rightarrow A_m^0 = \frac{N_{\gamma_0-1}(\frac{m}{\kappa})}{J_{\gamma_0-1}(\frac{m}{\kappa})}$.

p	0	1	2	3	4
$J(p)$	0.032	0.056	0.134	0.392	1.369

TABLE II: Numerical value of the integral (31) for different number of extra compact dimensions p .

Here the Casimir force, upon use of these modes, becomes

$$f_T = \frac{2}{3+p} \sum_{\{n\}} \left[\frac{\psi_0^2}{\kappa} f_{4D} \left(m_4 \Big|_{m=0} \right) + \int_0^\infty \frac{dm}{\kappa} \psi_m^2(0) f_{4D}(m_4) \right], \quad (26)$$

which can be read as saying that extra dimensional modes contribute to the Casimir force as individual discrete modes or as an admixture of discrete and continuous modes weighted by squared wave functions of the modes in the non compact direction on the brane.

In the low energy regime $m \ll \kappa$ and only the zero modes associated to the compact extra dimensions are relevant ($n_1 = \dots = n_p = 0$) so

$$A_m^0 \approx -\frac{\Gamma(\gamma_0)\Gamma(\gamma_0-1)}{\pi} \left(\frac{m}{2\kappa} \right)^{2-2\gamma}, \quad (27)$$

and the wave function of the continuous modes at the brane becomes

$$\psi_m^2(y \rightarrow 0) \approx \frac{1}{2^{p+1} \left[\Gamma\left(\frac{2+p}{2}\right) \right]^2} \frac{m^{p+1}}{\kappa^{p+1}}. \quad (28)$$

The Casimir force takes the form

$$f_{\text{eff}}^0 \approx \frac{2+p}{3+p} f_{4D}(m_4=0) + f_{\text{light}}^0, \quad (29)$$

$$f_{\text{light}}^0 := \frac{1}{2^p (3+p) \left[\Gamma\left(\frac{2+p}{2}\right) \right]^2} \frac{1}{\kappa^{2+p}} \int_{m \ll \kappa} m^{1+p} f_{4D}(m) dm,$$

with $f_{4D}(m)$ given by (19). The light modes contribution can be evaluated further as

$$f_{\text{light}}^0 \approx \frac{480}{\pi^2 2^p (3+p) \left[\Gamma\left(\frac{2+p}{2}\right) \right]^2} \frac{1}{(\kappa l)^{2+p}} f_{4D}(0) J(p), \quad (30)$$

where $J(p)$ is defined by

$$J(p) \equiv \frac{1}{32\pi^2} \int_0^\infty \alpha^{1+p} \int_{2\alpha}^\infty dx \frac{x^2 \sqrt{x^2 - 4\alpha^2}}{e^x - 1}. \quad (31)$$

In table II we give the value of this integral for p in the range $0, \dots, 4$.

Considering again $f_{\text{light}} \ll \frac{2+p}{3+p} f_{4D}(0)$ in (29) one gets the lower bound $(\kappa l)^{2+p} \gg 480 J(p) / 3 \pi^2 2^p \left[\Gamma\left(\frac{2+p}{2}\right) \right]^2$. By taking $l \sim 10^{-6}\text{m}$ of typical Casimir experiments one gets an upper bound for the anti de Sitter radius of $\kappa^{-1} \ll (3 \pi^2 2^p \left[\Gamma\left(\frac{2+p}{2}\right) \right]^2 / 480 J(p))^{1/(2+p)} \times 10^{-6}\text{m}$.

V. DISCUSSION

Casimir or vacuum force [26, 27, 28, 29, 30, 31, 32, 33] appears amongst a set of little explored low energy tests, possibly including high precision atomic experiments [21], amenable to probe brane worlds. Two of the reasons to consider it are the fact it depends on the mode structure of matter fields and also that experimentally it lies in the range of recent and future sub-millimeter gravity experiments [14, 15, 16], both of which are related to brane worlds models. With this motivation in this letter we have determined brane world corrections to standard 4D Casimir force for a scalar field subject to Dirichlet boundary conditions on two parallel plates lying within the single $(3+p)$ -brane of Randall-Sundrum type of scenario or RSII- p . We adopt the Green's function approach to do so and hence as a first step a study of the modes for the scalar field was developed in the background brane world.

The resulting scalar modes group themselves as a continuous tower of massive modes presenting a single quasilocalized massive mode or as a massless mode incorporated with a continuous tower of massive modes depending on whether the scalar field is massive or not. In the low energy approximation for either case only the zero modes corresponding to the compact dimensions and light modes for the non compact dimension are considered. They yield a Casimir force naturally splitting into a leading order term given by the quasilocalized or the massless mode, for the massive and massless scalar field, respectively, plus a correction term coming from the light continuous modes, Eqs. (22) and (29). Such leading order terms coincide with the standard 4D Casimir force for massive or massless scalar field, respectively, up to numerical factors depending on the number of compact dimensions p . Moreover corrections turn out to be attractive and depend on the separation between plates as $l^{-(10+p)}$ for the massive scalar field and as $l^{-(6+p)}$ for the massless one, Eqs. (23) and (30), respectively. To keep corrections negligible compared to the 4D force upper bound for $AdS_{(5+p)}$ radius κ^{-1} is obtained of $\sim 10^{-6}\text{m}$.

It is illustrative to compare the obtained Casimir force with others in recent literature. For $p = 1$ the results of [23] are recovered except for a numerical factor: $\frac{3}{4}$ instead of $\frac{3}{2}$ in Eqs. (22) and (29) due to the volume factor (12). Furthermore, the case $p = 0$ is of particular interest since it was studied in [46] by the dimensional regularization technique. It is worth stressing that this case fails to be an adequate model for the realistic electromagnetic case since for the latter localization does not work as opposed to the scalar field. Still, theoretically, its

study sheds light on the comparison between different approaches. In [46] they get

$$F_{Total} = f_{4D}(0) \left(1 + \frac{45}{2\pi^3} \zeta(5) \frac{1}{l\kappa} \right), \quad (32)$$

which clearly differs from ours, Eqs. (29)-(30) for $p = 0$. We now comment on this difference. It was shown in [52] that the Casimir force per unit area on the boundary plates for a scalar field in a spacetime with d Euclidean spatial dimensions is always attractive and is given by

$$f_d = -\frac{d}{l^{d+1}} \Gamma\left(\frac{d+1}{2}\right) (4\pi)^{-(d+1)/2} \zeta(d+1). \quad (33)$$

If we sum the Casimir force for a massless scalar field in 4D ($d=3$) with the Casimir force for a massless scalar field in 5D ($d=4$) we obtain

$$F = f_{4D} \left(1 + \frac{45}{4\pi^4} \zeta(5) \frac{1}{l} \right). \quad (34)$$

Comparing (32) with (34) we see that these equations differ only by a factor $2\pi/\kappa$ in the second term which can be understood as follows. The modified dispersion relations used in obtaining (32) have an extra continuous mass term m^2 with $m \in [0, \infty)$, therefore when computing the force there is a term with an extra integral $\int dm/\kappa = (2\pi/\kappa) \int dm/2\pi$, which amounts to have $(2\pi/\kappa)f_{d=4}$ that corresponds to $2\pi/\kappa$ times the force for a 5D scalar field in flat space. Interpreting (32) now is clear: it contains two terms, the standard 4D Casimir force for a massless scalar field in Minkowski spacetime plus $2\pi/\kappa$ times the Casimir force for a 5D massless scalar field in 5D Minkowski spacetime. Indeed dimensional regularization as considered above is blind to physical information like the curvature of the background. Such state of affairs could be remedied by considering dimensional regularization on curved backgrounds [55]. Then both approaches could be fairly compared.

Future problems suggest themselves based on the previous analysis. For instance, among several other models, it would be interesting to obtain the effective Casimir force for a scalar field in the two branes RSI background metric, possible extended by compact dimensions, using Green's function method to contrast it with that obtained by dimensional regularization in [46].

Finally, a comment regarding boundary conditions is in order. The scalar modes have been built to fulfil Dirichlet boundary conditions in ordinary 3-space, to implement the presence of the plates here represented by two planes. Along the extra dimensions the boundary conditions used were those appropriate for the background brane model: matching

along the brane for the non compact dimension and periodic for the compact ones. Hence, as stressed in this work, we have plates extended along extra dimensions which, however -and this is crucial physically- appear as two parallel planes within ordinary 3-space. Indeed our Casimir setting works as shown by the leading order terms corresponding to those in 4D, Eqs. (22) and (29). Moreover it generalizes to brane worlds of type RSII- p previous results for arbitrary number of flat dimensions with two codimension one plates [52]. It would be interesting to study the situation in which one restrains the plates to 3-space to coincide with two planes without stretching along extra dimensions. They would have dimension 2 within $3+1+p$ space, thus having codimension $2+p$ (see for instance [56]).

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- [1] G. Nordstrom, Phys. Z. **15**, 504 (1914), physics/0702221.
 - [2] T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) **1921**, 966 (1921).
 - [3] O. Klein, Z. Phys. **37**, 895 (1926).
 - [4] K. Akama, Lect. Notes Phys. **176**, 267 (1982), hep-th/0001113.
 - [5] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **B125**, 136 (1983).
 - [6] M. Visser, Phys. Lett. **B159**, 22 (1985), hep-th/9910093.
 - [7] P. Horava and E. Witten, Nucl. Phys. **B475**, 94 (1996), hep-th/9603142.
 - [8] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996), hep-th/9510209.
 - [9] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999), hep-th/9906064.
 - [10] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999), hep-ph/9905221.
 - [11] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. **B436**, 257 (1998), hep-ph/9804398.
 - [12] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. **B429**, 263 (1998), hep-

- ph/9803315.
- [13] I. Antoniadis, Phys. Lett. **B246**, 377 (1990).
 - [14] C. D. Hoyle et al., Phys. Rev. Lett. **86**, 1418 (2001), hep-ph/0011014.
 - [15] E. G. Adelberger et al., Phys. Rev. Lett. **98**, 131104 (2007), hep-ph/0611223.
 - [16] D. J. Kapner et al., Phys. Rev. Lett. **98**, 021101 (2007), hep-ph/0611184.
 - [17] V. A. Rubakov, Phys. Usp. **44**, 871 (2001), hep-ph/0104152.
 - [18] F. Feruglio, Eur. Phys. J. **C33**, s114 (2004), hep-ph/0401033.
 - [19] R. Maartens, Living Rev. Rel. **7**, 7 (2004), gr-qc/0312059.
 - [20] A. Pérez-Lorenzana, J. Phys. Conf. Ser. **18**, 224 (2005), hep-ph/0503177.
 - [21] R. Bluhm, AIP Conf. Proc. **539**, 109 (2000), hep-ph/0006033.
 - [22] F. Luo and H. Liu, Int. J. Theor. Phys. **46**, 606 (2007), gr-qc/0602093.
 - [23] R. Linares, H. A. Morales-Técotl, and O. Pedraza, Phys. Rev. **D77**, 066012 (2008), arXiv:0712.3963 [hep-ph].
 - [24] H. A. Morales-Técotl, O. Pedraza, and L. O. Pimentel, Gen. Rel. Grav. **39**, 1185 (2007), physics/0611241.
 - [25] R. Linares, H. A. Morales-Técotl, and O. Pedraza, Phys. Lett. **B633**, 362 (2006), hep-ph/0505109.
 - [26] H. B. G. Casimir, Kon. Ned. Akad. Wetensch. Proc. **51**, 793 (1948).
 - [27] M. J. Sparnaay, Physica **24**, 751 (1958).
 - [28] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998).
 - [29] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002), quant-ph/0203002.
 - [30] R. S. Decca, *et al.*, Annals Phys. **318**, 37 (2005), quant-ph/0503105.
 - [31] G. L. Klimchitskaya, *et al.*, Int. J. Mod. Phys. **A20**, 2205 (2005), quant-ph/0506120.
 - [32] S. K. Lamoreaux, Rept. Prog. Phys. **68**, 201 (2005).
 - [33] K. A. Milton, *The Casimir effect: Physical manifestations of zero-point energy* (River Edge, USA: World Scientific, 2001).
 - [34] M. Fabinger and P. Horava, Nucl. Phys. **B580**, 243 (2000), hep-th/0002073.
 - [35] H. Gies, K. Langfeld, and L. Moyaerts, JHEP **06**, 018 (2003), hep-th/0303264.
 - [36] I. Brevik and A. A. Bytsenko (2000), hep-th/0002064.
 - [37] L. Hadasz, G. Lambiase, and V. V. Nesterenko, Phys. Rev. **D62**, 025011 (2000), hep-

- th/9911248.
- [38] E. Elizalde, S. Nojiri, S. D. Odintsov, and S. Ogushi, Phys. Rev. **D67**, 063515 (2003), hep-th/0209242.
 - [39] J. Garriga and A. Pomarol, Phys. Lett. **B560**, 91 (2003), hep-th/0212227.
 - [40] O. Pujolas, Int. J. Theor. Phys. **40**, 2131 (2001), hep-th/0103193.
 - [41] A. Flachi and D. J. Toms, Nucl. Phys. **B610**, 144 (2001), hep-th/0103077.
 - [42] W. D. Goldberger and I. Z. Rothstein, Phys. Lett. **B491**, 339 (2000), hep-th/0007065.
 - [43] S. Nojiri, S. D. Odintsov, and S. Zerbini, Class. Quant. Grav. **17**, 4855 (2000), hep-th/0006115.
 - [44] R. Durrer and M. Ruser, Phys. Rev. Lett. **99**, 071601 (2007), 0704.0756.
 - [45] M. Ruser and R. Durrer, Phys. Rev. **D76**, 104014 (2007), 0704.0790.
 - [46] M. Frank, I. Turan, and L. Ziegler, Phys. Rev. **D76**, 015008 (2007), arXiv:0704.3626 [hep-ph].
 - [47] S. L. Dubovsky, V. A. Rubakov, and P. G. Tinyakov, Phys. Rev. **D62**, 105011 (2000), hep-th/0006046.
 - [48] S. L. Dubovsky, V. A. Rubakov, and P. G. Tinyakov, JHEP **08**, 041 (2000), hep-ph/0007179.
 - [49] I. Oda, Phys. Lett. **B496**, 113 (2000), hep-th/0006203.
 - [50] T. Gherghetta and M. E. Shaposhnikov, Phys. Rev. Lett. **85**, 240 (2000), hep-th/0004014.
 - [51] T. Gherghetta, E. Roessl, and M. E. Shaposhnikov, Phys. Lett. **B491**, 353 (2000), hep-th/0006251.
 - [52] J. Ambjorn and S. Wolfram, Ann. Phys. **147**, 1 (1983).
 - [53] B. Bajc and G. Gabadadze, Phys. Lett. **B474**, 282 (2000), hep-th/9912232.
 - [54] S. B. Giddings, E. Katz, and L. Randall, JHEP **03**, 023 (2000), hep-th/0002091.
 - [55] E. Elizalde, J. Phys. **A39**, 6299 (2006), hep-th/0607185.
 - [56] A. Scardicchio, Phys. Rev. **D72**, 065004 (2005), hep-th/0503170.